EE 2240

Problem #07

Use Cramer's Rule to solve for V_x.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -0.2 & 0 \\ -1/45 & 17/225 & -1/30 & -1/50 & -0.02 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1/50 & 0 & -1/50 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_x \\ V_y \\ I_y \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since V_x is the 5th variable, replace the 5th column of the coefficient matrix with the input vector from the right-hand side, and the calculate the determinant of the resulting matrix. V_x is the ratio of this result to the determinant of the original coefficient matrix.

Numerator =

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{5} & 0 \\ -\frac{1}{45} & \frac{17}{225} & -\frac{1}{30} & -\frac{1}{50} & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{1}{50} & 0 & -\frac{1}{50} & 0 & 0 & 1 \end{vmatrix}$$

(continued on next page)

$$= -100 \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & -1/5 & 0 \\ -1/45 & 17/225 & -1/30 & -1/50 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1/50 & 0 & -1/50 & 0 & 1 \end{vmatrix}$$

$$= -100(1) \begin{vmatrix} 0 & 0 & 1 & -1/5 & 0 \\ 17/225 & -1/30 & -1/50 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1/50 & 0 & -1/50 & 0 & 1 \end{vmatrix} - 100(5) \begin{vmatrix} 0 & 0 & 0 & 1 & -1/5 \\ -1/45 & 17/225 & -1/30 & -1/50 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 1/50 & 0 & -1/50 & 0 \end{vmatrix}$$

$$= -100(1) \begin{vmatrix} 0 & 0 & 1 & -1/5 \\ 17/225 & -1/30 & -1/50 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix} - 500\left(\frac{1}{45}\right) \begin{vmatrix} 0 & 0 & 1 & -1/5 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 1 \\ 1/50 & 0 & -1/50 & 0 \end{vmatrix}$$

$$= -100 \left\{ 1 \begin{vmatrix} 17/225 & -1/30 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} + \frac{1}{5} \begin{vmatrix} 17/225 & -1/30 & -1/50 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{vmatrix} \right\} - \frac{100}{9} \left\{ 1 \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1/50 & 0 & 0 \end{vmatrix} + \frac{1}{5} \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1/50 & 0 & -1/50 \end{vmatrix} \right\}$$

(continued on next page)

$$= -100\left\{\frac{17}{225} + \frac{1}{5}\left(-\frac{17}{225} + \frac{1}{30} + \frac{1}{50}\right)\right\} - \frac{100}{9}\left\{\frac{1}{50} + \frac{1}{5}\left(-\frac{1}{50} + \frac{1}{50}\right)\right\}$$

$$= -\frac{3300}{450}$$

$$= -\frac{22}{3}$$

Denominator =

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1/5 \\ -1/45 & 17/225 & -1/30 & -1/50 & -1/50 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 1/50 & 0 & -1/50 & 0 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 0 & 0 & 0 & -1/5 \\ 17/225 & -1/30 & -1/50 & -1/50 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1/50 & 0 & -1/50 & 0 & 0 \end{vmatrix} - 5\begin{vmatrix} 0 & 0 & 1 & 0 & -1/5 \\ -1/45 & 17/225 & -1/30 & -1/50 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 1/50 & 0 & -1/50 & 0 \end{vmatrix}$$

$$= 1(-1)\begin{vmatrix} 0 & 1 & 0 & -1/5 \\ 17/225 & -1/50 & -1/50 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1/50 & -1/50 & 0 & 0 \end{vmatrix} - 5(1)\begin{vmatrix} 0 & 1 & 0 & -1/5 \\ -1/45 & -1/50 & -1/50 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1/50 & -1/50 & 0 & 0 \end{vmatrix}$$

(continued on next page)

$$= -1(1)\begin{vmatrix} 0 & 1 & 0 & -1/5 \\ 17/225 & -1/50 & -1/50 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{vmatrix} - 5\left(\frac{1}{45}\right)\begin{vmatrix} 0 & 1 & 0 & -1/5 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1/50 & -1/50 & 0 & 0 \end{vmatrix}$$

$$= -1\left\{1\begin{vmatrix} 0 & 0 & -1/5 \\ 17/225 & -1/50 & 0 \\ 1 & 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 & -1/5 \\ 17/225 & -1/50 & 0 \\ 1 & -1 & 1 \end{vmatrix}\right\} - \frac{1}{9}(-1)\begin{vmatrix} 0 & 1 & -1/5 \\ 1 & -1 & 1 \\ 1/50 & -1/50 & 0 \end{vmatrix}$$

$$= -\left\{-\frac{1}{250} + \left(\frac{17}{1125} - \frac{1}{250} - \frac{17}{225}\right)\right\} + \frac{1}{9}\left(\frac{1}{50} + \frac{1}{250} - \frac{1}{250}\right)$$

$$= \frac{159}{2250}$$

$$= \frac{53}{750}$$

$$V_x = \frac{\text{Numerator}}{\text{Denominator}} = \frac{-22/3}{53/750} = -\frac{5500}{53} \approx -103.77$$